At Michaelchurch Escley Primary School the children receive a daily maths lesson. As a basis for planning, the staff use the Program of Study for Maths from the 2013 National Curriculum. This outlines what is expected of each child for each year group. This leaflet outlines the methods used by the staff to aid the teaching and learning of written calculations. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. The children will use the methods taught, and apply them to a wide variety of problem solving situations.

With all the methods demonstrated here, accurate and clear presentation, making correct use of the squared pages in a maths book for example, is vital for success.

January 2016
Written Methods for Addition

To add successfully, children need to be able to:
• recall all addition pairs to 9 + 9 and know by heart pairs of numbers that make 10
• add mentally a series of one-digit numbers, such as 5 + 8 + 4
• add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
• partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways

The following examples show the developmental stages of methods the children will be taught in order to use an efficient written method for addition of whole numbers.

Informal methods

Jottings
In the first instance, children will add by combining groups using concrete objects. From this they will progress to drawing the groups.
Eg.

Jane had 3 bears. She was given 2 more. How many does she have now?
Structured Number-lines

As children gain confidence with the number system, they progress to using number-lines and 100 squares to count on. Eg.

Un-structured Number-lines

The limitations of structured number-lines (a 1 – 20 number-line is no good for solving 48 + 36) demand a new strategy. Eg.

This method demands a confident understanding of place value for it to be a valuable calculation strategy.
Formal Methods

Column Addition

The children will develop column addition of the least significant digit (the units) leading to a standard written method. Carry digits are recorded below the line and the children will be encouraged to use the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’.

\[
\begin{array}{c}
358 \\
+ 73 \\
\hline
11 \\
\hline
120 \\
\hline
300 \\
\hline
431
\end{array}
\]

leading to ‘carrying’ below the line

\[
\begin{array}{c}
625 \\
+ 783 \\
\hline
367 \\
\hline
673 \\
+ 825 \\
\hline
852
\end{array}
\]

The children will extend their use of this method to any numbers and to decimals.

\[
\begin{array}{c}
587 \\
+ 475 \\
\hline
1062 \\
\hline
\end{array}
\quad
\begin{array}{c}
3587 \\
+ 675 \\
\hline
4262 \\
\hline
\end{array}
\quad
\begin{array}{c}
7648 \\
+ 1486 \\
\hline
9134 \\
\hline
\end{array}
\quad
\begin{array}{c}
6584 \\
+ 5848 \\
\hline
12432 \\
\hline
\end{array}
\]

£5.75
+£3.18
£8.93

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\hline
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\quad
\begin{array}{c}
6584 \\
+ 5848 \\
\hline
12432 \\
\hline
\end{array}
\]

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Written Methods for Subtraction

To subtract successfully, children need to be able to:
• recall all addition and subtraction facts to 20
• subtract multiples of 10 (such as 160 – 70) using the related subtraction fact, 16 – 7, and their knowledge of place value
• partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)

The following examples show the developmental stages of methods the children will be taught in order to use an efficient method for subtraction of two-digit and three-digit whole numbers.

Informal methods

Jottings
In the first instance, children will subtract by physically taking away from groups using concrete objects. From this they will progress to drawing the groups.

Eg.

There were 8 balloons and 2 popped, how many were left?
Structured Number-lines

As children gain confidence with the number system, they progress to using number-lines and 100 squares to count back.

Eg.

![Number-line example](image)

6 - 3 = 3

Un-structured Number-lines

The limitations of structured number-lines (a 1 – 20 number-line is no good for solving 148 - 36) demand a new strategy. We teach children to draw their own number-line. They are also taught to decide if counting on or back is the best strategy. Generally it is easier to count on but if the difference between the 2 numbers is very large, counting back can be a more efficient option.

Eg.

84 − 56 = 28

4 + 20 + 4 = 28

783 − 356

4 + 40 + 300 + 83 = 427
Formal method

Expanded layout leading to the standard method of decomposition

Partitioning the numbers into tens and ones and writing one under the other mirrors the standard column method where ones are placed under ones and tens under tens. This expanded method leads children to the more compact method so that they understand its structure and efficiency.

The amount of time spent practising this expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

The children will also extend their use of this method to include calculating with decimals.

Example: 74 – 27

\[
\begin{align*}
70 &+ 4 \\
-20 &+ 7 \\
\hline
40 &+ 7
\end{align*}
\]

\[
\begin{align*}
60 &+ 14 \\
-20 &+ 7 \\
\hline
46 &+ 14
\end{align*}
\]

Dealing with zeros when adjusting

\[
\begin{align*}
500 &+ 0 + 3 \\
-200 &+ 70 + 8 \\
\hline
200 &+ 70 + 5
\end{align*}
\]

\[
\begin{align*}
400 &+ 90 + 13 \\
-200 &+ 70 + 8 \\
\hline
200 &+ 20 + 5
\end{align*}
\]
Written Methods for Multiplication

To multiply successfully, children need to be able to:
• recall all multiplication facts to $10 \times 10$
• partition number into multiples of one hundred, ten and one
• work out products such as $70 \times 5$, $70 \times 50$, $700 \times 5$ or $700 \times 50$
  using the related fact $7 \times 5$ and their knowledge of place value
• add two or more single-digit numbers mentally
• add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value
• add combinations of whole numbers using the column method.

The following examples show the developmental stages of methods the children will be taught in order to use an efficient written method for multiplication.

Informal methods

Jottings
In the first instance, children will multiply by physically combining groups (lots of) using concrete objects and counting the total number of objects. From this they will progress to drawing the groups.

\[ 5 \times 3 = 15 \]
Repeated Addition and Arrays

It is important for children to understand multiplication as:
- **repeated addition**: for example:
  5 added together 3 times is $5 + 5 + 5$, or 3 lots of 5, or 3 times 5, or $5 \times 3$ (or $3 \times 5$)

This can initially be represented pictorially:

![Pictorial representation of repeated addition](image)

Children will then progress to using structured number-lines:

![Structured number-line example](image)

For larger numbers, children could draw their own (unstructured) number-lines.
Eg.

$$10 \times 5 = 50$$
• describing an array: for example,

```
■ ■ ■ ■ ■  4 \times 2 = 8
■ ■ ■ ■ ■
2 \times 4 = 8
```

As the development of multiplication facts is vital for all future learning, children will need to become confident with mental strategies before beginning to develop a written method for multiplication.

*Mental multiplication using partitioning*

Children will record mental multiplication using partitioning.

\[
14 \times 3 = (10 + 4) \times 3 \\
= (10 \times 3) + (4 \times 3) = 30 + 12 = 42
\]

\[
43 \times 6 = (40 + 3) \times 6 \\
= (40 \times 6) + (3 \times 6) = 240 + 18 = 258
\]
The grid method

The grid method enables children to partition a number and then carry out the multiplication. This allows the children to have a clear understanding of the calculations involved in the problem.

For example, $23 \times 8$ is approximately $20 \times 10 = 200$.

\[
\begin{array}{c|c|c|c}
& 20 & 3 \\
\hline
8 & 160 & 24 \\
\hline
& 160 + 24 \\
\hline
& 184 \\
\end{array}
\]

The children will develop their understanding of this method by applying it to larger numbers as shown below.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
& 300 & 40 & 6 \\
\hline
346 & 2700 & 360 & 54 \\
\hline
& 2700 + 54 \\
\hline
& 3114 \\
\end{array}
\]

$72 \times 38$ is approximately $70 \times 40 = 2800$

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
& 70 & 2 \\
\hline
72 & 2100 & 60 \\
\hline
& 560 + 16 \\
\hline
& 2736 \\
\end{array}
\]
Formal written methods

*Short multiplication*

The children will use a more expanded version of vertical recording based on the grid method. The recording is reduced further with carry digits recorded below the line.

\[
\begin{array}{c}
\begin{array}{c}
23 \\
\times 7 \\
\end{array}
\end{array}
\begin{array}{c}
\frac{140}{(20 \times 7)} \\
\frac{21}{(3 \times 7)}
\end{array}
\begin{array}{c}
\frac{161}{2}
\end{array}
\]

leading to

\[
\begin{array}{c}
\begin{array}{c}
23 \\
\times 7 \\
\end{array}
\end{array}
\begin{array}{c}
\frac{161}{2}
\end{array}
\]

\[
4346 \times 8 \text{ is approximately } 4500 \times 10 = 45,000.
\]

\[
\begin{array}{c}
4346 \times 8 \\
\times 4000 300 40 6
\end{array}
\begin{array}{c}
32000 \\
2400 \\
320 \\
+ 48 \\
\end{array}
\begin{array}{c}
34768
\end{array}
\]

\[
372 \times 24 \text{ is approximately } 400 \times 20 = 8000
\]

\[
\begin{array}{c}
372 \times 24 \\
\times 300 70 2
\end{array}
\begin{array}{c}
6000 \\
1400 \\
1200 \\
280 \\
40 \\
+ 8 \\
\end{array}
\begin{array}{c}
8928
\end{array}
\]
The children will develop their understanding of this method by applying it to larger numbers as shown below and to decimals.

\[
\begin{array}{c}
346 \\
\times 9
\end{array}
\]

\[
\begin{array}{c}
2700 \\
(300 \times 9)
\end{array}
\]

\[
\begin{array}{c}
360 \\
(40 \times 9)
\end{array}
\]

\[
\begin{array}{c}
54 \\
(6 \times 9)
\end{array}
\]

\[
\begin{array}{c}
3114
\end{array}
\]

\[
\begin{array}{c}
346 \\
\times 9
\end{array}
\]

\[
\begin{array}{c}
3114
\end{array}
\]

\[
\begin{array}{c}
4346 \\
\times 8
\end{array}
\]

\[
\begin{array}{c}
32000 \\
(4000 \times 8)
\end{array}
\]

\[
\begin{array}{c}
2400 \\
(300 \times 8)
\end{array}
\]

\[
\begin{array}{c}
320 \\
(40 \times 8)
\end{array}
\]

\[
\begin{array}{c}
48 \\
(6 \times 8)
\end{array}
\]

\[
\begin{array}{c}
34768
\end{array}
\]

\[
\begin{array}{c}
4.92 \\
\times 3
\end{array}
\]

\[
\begin{array}{c}
12.00 \\
(4.00 \times 3)
\end{array}
\]

\[
\begin{array}{c}
2.70 \\
(0.90 \times 3)
\end{array}
\]

\[
\begin{array}{c}
0.06 \\
(0.02 \times 3)
\end{array}
\]

\[
\begin{array}{c}
14.76
\end{array}
\]

\[
\begin{array}{c}
4.92 \\
\times 3
\end{array}
\]

\[
\begin{array}{c}
14.76
\end{array}
\]
Long multiplication

Children will apply their understanding of this method to long multiplication of TU x TU and HTU x TU as shown below.

72 x 38 is approximately 70 x 40 = 2800.

\[
\begin{array}{c}
72 \\
\times 38 \\
\hline
2160 \\
576 \\
\hline
2736
\end{array}
\]

(72 x 30) (72 x 8)

352 x 27 is approximately 350 x 30 = 10,500.

\[
\begin{array}{c}
352 \\
\times 27 \\
\hline
7040 \\
2464 \\
\hline
9504
\end{array}
\]

(352 x 20) (352 x 7)
Written Methods for Division

To carry out written methods of division successfully, children need to be able to:
• understand division as repeated subtraction;
• estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
• multiply a two-digit number by a single-digit number mentally;
• subtract numbers using the column method.

The following examples show the developmental stages of methods the children will be taught in order to use an efficient written method for division.

Informal Methods

_Jottings_

In the first instance, children will multiply by physically combining groups (lots of) using concrete objects and counting the total number of objects. From this they will progress to drawing the groups.

\[45 \div 5 = 9\]
Number-lines – structured and unstructured

Children will then progress to using structured number-lines:

\[ 15 \div 5 = 3 \]

---

Subtracting multiples of the divisor or ‘chunking’

This method, often referred to as ‘chunking’, is based on subtracting multiples of the divisor, or ‘chunks’ from the number to be divided.

\[
\begin{align*}
72 \div 5 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 72 \\
\quad - 50 \quad \quad \quad \quad \quad \quad \quad \quad (10 \times 5) \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 22 \\
\quad - 20 \quad \quad \quad \quad \quad \quad \quad \quad (4 \times 5) \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 2
\end{align*}
\]

Answer 14 remainder 2
The children will develop their understanding of this method by applying it to larger numbers as shown below.

\[
\begin{array}{r}
256 \div 7 & \quad 256 \\
- 70 & \quad (10 \times 7) \\
\hline
186 \\
- 140 & \quad (20 \times 7) \\
\hline
46 \\
- 42 & \quad (6 \times 7) \\
\hline
4 \\
\end{array}
\]

Answer: 36 remainder 4

The key to the efficiency of chunking lies in the estimate that is made before the chunking starts enabling children to carry out the calculation in the smallest amount of steps.

\[
\begin{array}{r}
256 \div 7 & \quad 256 \\
- 210 & \quad (30 \times 7) \\
\hline
46 \\
- 42 & \quad (6 \times 7) \\
\hline
4 \\
\end{array}
\]

Answer: 36 remainder 4
The children may then apply their understanding of this method to decimals.

$$7) \quad 87.5$$

- $70.0$ \hspace{1cm} (10 x 7)

\hspace{1cm} 17.5

\hspace{1cm} 14.0 \hspace{1cm} (2 x 7)

\hspace{1cm} 3.5

\hspace{1cm} 3.5 \hspace{1cm} (0.5 \times 7)

\hspace{1cm} 0.0

**Answer:** 12.5

**Formal written methods**

*Short Division or ‘The Bus Stop’ method*

432 ÷ 5 becomes

4 ÷ 5 you can’t do
so try 43 ÷ 5
which= 8

8 x 5 = 40 so there
is a remainder of 3
which you carry
across

32 ÷ 5 = 6
6 x 5 = 30
32 – 30 = 2

**Answer:** 86 remainder 2
Long division

The next stage is to tackle long division. This is an excellent strategy but takes a lot of practice to master. Its beauty rests in the extremely simple calculations that are generally required for each stage of the process. It is also very easy to express answers as decimals using this method.

\[ 432 \div 15 \text{ becomes} \]

\[
\begin{array}{c}
15 \longdiv{432} \\
\underline{30} \\
132 \\
\underline{120} \\
\underline{120} \\
0
\end{array}
\]

Answer: 28.8